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THE DIVISION OF A CIRCLE OR SPHERICAL SURFACE INTO EQUAL-AREA CELLS OR PIXELS

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The need to partition a circular area into many equally sized cells or pixels had arisen in several problems dealing with mapping phenomena within a circle. A radar picture of the horizontal extent of precipitation surrounding a station is usually of the circular area surrounding the station within approximately 192 kilometers 120 miles) radius. The radar picture is conveniently analyzed bye the presence or absence of a radar echo in each small cell within the circle. Since the spherical surface of the earth can be portrayed on a equal-are map, the proposed method of dividing the circle into equally sized pixels is especially applicable for detailed mapping of a spherical surface such as the northern hemisphere.

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The Division of a Circle or Spherical Surface Into Equal-Area Cells or Pixels

1. INTRODUCTION

The task is to divide a circular area, such as a PPI-radar scope picture, into equal units of area or cells. Although the units could be few and large, generally an area must be partitioned into many small cells or pixels. If the pixels can be made sufficiently small, then mapping a phenomenon such as areal rainfall becomes a matter of assigning a single value (for example, rainfall amount) to each pixel.

A flat circle can be used for mapping properties such a clouds on a sky dome, or properties on the spherical surface of the earth. [1] Rossow and Garder addressed this problem and developed and approximate solution for the equal-area net grid.[2]

The procedure presented in this report, however, yields exact equal-area pixels.

2. METHOD

The proposed method begins by dividing the circle (Figure 1) of overall radius R into a central unit circle of radius δr and M rings, each ring of thickness $2\delta r$, where

$$(2M + 1)\delta r = R \tag{1}$$

The unit area δA is that of the central cell

$$\delta A = \pi(\delta r)^2. \tag{2}$$

The circle that includes the first ring has radius $3\delta r$, and the area A_1 within this circle is given by $A_1 = 9\pi(\delta r)^2$.

The area in the first ring, or annulus, $\delta A_1 = 8\pi(\delta r)^2$. If the ring is now divided into 8 equal cells, each cell will be equal in area to the central unit cell δA.

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[1]Gringorten, Irving (1981) *Mapping the Climate* Environmental Research Papers, No. 723. Air Force Geophysics Laboratory, AFGL-TR-81-0015 (AD-A102904)

[2]Rossow, W.B., and Garder, L. (1984) Section of a Map Grid for Data Analysis and Archival. *Journal of Climate and Applied Meteorology*, **23**:1253-1257.

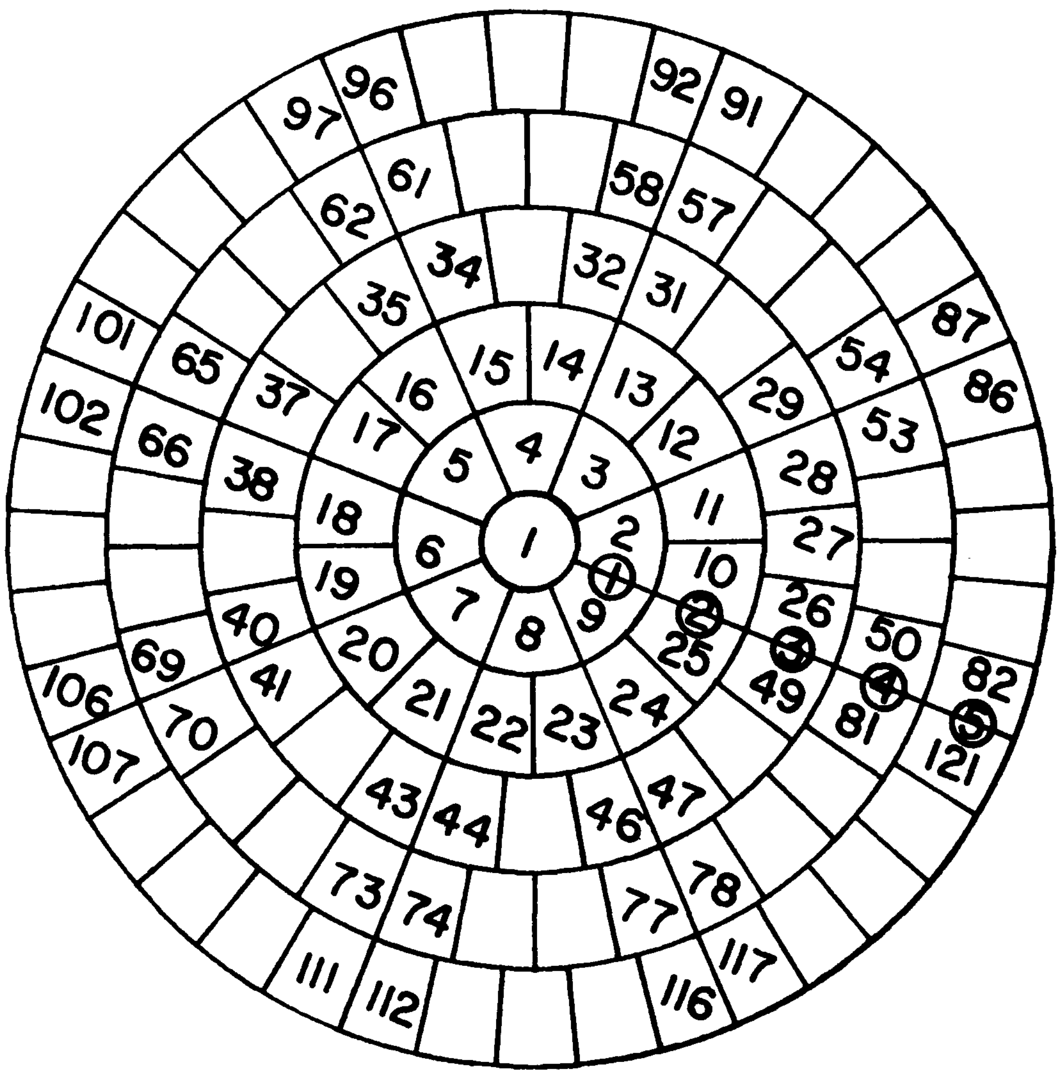

Figure 1: Division of a Circular Area into Circles and Then Equal-Area Pixels, Showing Numbering of Pixels

Likewise, the second ring will have outer radius $5\delta r$ and it can be divided into 2×8 cells, each having area δA. Similarly the mth ring can be divided into $8m$ cells, each having the area δA. The total number of cells I_m up to and including the mth ring will be

$$I_m = (2m + 1)^2 \tag{3}$$

for a total area A_m of

$$A_m = (2m + 1)^2 \pi (\delta r)^2. \tag{4}$$

Figure 1 depicts the division of a circle into five rings, for a total of 121 equal-area cells. The shapes of the cells begin with the small circle at the center: cell number 1. In the first ring the 8 cells are shaped like pieces of pie with the tips eaten away. In the remaining rings the shapes of the elementary units converge onto the rectangular shape of length $2\delta r$ and width $\pi(\delta r)/2$.

The cells are numbered, beginning with the number 1 for the central cell, and increasing by $8m$ with the mth ring ($m = 1, 2, 3, ...$).

Problem 1: *To find a cell number I, given the polar coordinates (r, θ), when the incremental length δr is known.*

The cell number I is determined as follows:

Where the symbol $INT(x)$ denotes the integer part of the number x always less than x), then the number of the last whole ring n is given by

$$n = INT(n') \tag{5}$$

and n' is defined by

$$(2n' + 1)\delta r = r. \tag{6}$$

The number of the ring in which (r, θ) is located is $(n + 1)$. The total number of cells I_r in the n rings is

$$I_r = (2n + 1)^2. \tag{7}$$

The number of cells I_θ remaining in the $(n + 1)th$ ring is

$$I_\theta = INT(I'_\theta) + 1 \tag{8}$$

where

$$I'_\theta = (theta/2\pi)(8n + 8) \tag{9}$$

if θ is in radians, and $= (\theta/360)(8n + 8)$ if θ is in degrees.

Finally the cell number I in which (r, θ) is located is:

$$I = I_r + I_\theta \tag{10}$$

Problem 2: *To find the polar coordinates (r, θ) of the middle of a cell, given the cell number I and the incremental unit of length δr.*

The number of whole rings n is:

$$n = INT(n') \tag{11}$$

where n' is given by

$$(2n' + 1)^2 = I \tag{12}$$

(Note: If n' is a whole number then make $n = n' - 1$).

The radial distance r to the central point of the cell is

$$r = (2n + 2)\delta r \tag{13}$$

The number of cells I_r up to, and including, the nth ring is

$$I_r = (2n + 1)^2 \tag{14}$$

Hence the remaining number of cells I_θ is

$$I_\theta = I - I_r \tag{15}$$

which places the central point of the last cell at angle θ such that

$$\theta/2\pi = (I_\theta - 1/2)/(8n + 8) \tag{16}$$

if θ is in radians.

If angular measurements are made in degrees, the

$$\theta/360 = (I_\theta - 1/2)(8n + 8). \tag{17}$$

Equations (13), (16), or (17) provide the coordinates (r, θ).

Problem 3: *Application to the earth's northern hemisphere.*

This procedure can be applied readily to a spherical surface which can be projected onto a flat circular equal-area map. For the northern hemisphere the map's center represents the north pole and the outer perimeter represents the equator. In the Lambert Azimuthal Equal-area projection the parallels of latitude appear as concentric circles centered at the north pole, and the meridians of longitude appear as straight lines radiating outward from the center.

Let R_e be the radius of the spherical earth. That is,

$$R_e = 3437.75\text{nm} = 6371.00\text{km}.$$

The area of the northern hemisphere A is

$$A = 2\pi R_e^2. \tag{18}$$

The equal-area map (given a scale factor of 1.0) must have an overall radius R such that

$$R = R_e\sqrt{2} \tag{19}$$

If we choose to divide the circular map into M rings, the incremental value δr is defined by

$$(2M+1)\delta r = R. \tag{20}$$

The mth ring will have and outer radius r_m such that

$$r_m = (2M+1)\delta r. \tag{21}$$

The map radius r that corresponds to a parallel of latitude ϕ is:

$$r = R_e\sqrt{2(1-\sin\phi)}. \tag{22}$$

The map angular coordinate θ that corresponds to a meridian of longitude λ. is for positive longitude (east),

$$\theta = \lambda \tag{23}$$

for negative longitude (west), $\theta = \lambda + 2\pi$ (in radians), and $\theta = \lambda + 360$ (in degrees).

The division of the northern hemisphere into equal cells, and the numbering of the cells, proceeds as in the previous section.

Suppose the northern hemisphere is to be divided by parallels of latitude to make five rings $M = 5$, each ring with a width approximately 18°. Then the total number of cells I will be 121 [Eq (3)]. The incremental length δr on the corresponding equal-area map [Eq (20)] is:

$$\delta r = 819.08\text{km} = 441.97\text{nm}. \tag{24}$$

The latitudinal circles that divide the hemisphere into the five rings will correspond [Eq (20)] to the map radii r:

$$r = m(\delta r)$$

for m = 1, 3, 5, 7, 9, 11, for which Eq. (22) gives the parallels:

$$\phi = 82.63°, 67.76°, 52.50°, 36.52°, 19.30°, 0(equator)$$

For the location of Boston $\phi = 42.5°$, $\lambda = -71.0°$, Eqs (22) and (23) give $r = 2769$nm, $\theta = 289°$. With the given polar map coordinates (r, θ), Eqs (5) and (6) will provide the ring number $(n + 1 = 3)$ and Eqs (7), (8), (9), and (10) will provide the cell number $(I = 45)$.

Problem 4: *Climatology using equal-area cells.*

Suppose it is appropriate to study the climate in the northern hemisphere in very small pixels, such as $(5 \text{ km})^2$. The approximate number of pixels I' will be given by the total area of the northern hemisphere divided by the elemental area (25 km^2):

$$I' = 10,187,524.$$

The number m of rings is rounded to 1595 [Eq. (3)]. This, in turn, modifies the total number of cells to

$$I = 10,182,481$$

so that the area [Eq. (2)] of each cell δA is recalculated to be 25.01km^2. The incremental length δr is given by Eq. (2):

$$\delta r = 2.822\text{km}$$

For a point given by latitude and longitude, its cell number can be determined using Eqs (22), (23), (5), (6), (7), (8), and (10). Information can be stored in cells and referenced by the cell numbers.

3. CONCLUSION

The complexity involved in dividing a circle into equal-area pixels is warranted by the applications of such a technique. It can be applied to the earth's spherical surface, and also to a sky dome. The mapping technique provides a method for the equal division of the earth's surface in preparation for areal analysis around a central point, usually the north pole.

References

1. Gringorten, Irving (1981) *Mapping the Climate* Environmental Research Papers, No. 723, Air Force Geophysics Laboratory, AFGL-TR-81-0015 (AD-A102904).

2. Rossow, W.B., and Garder, L. (1984) Section of a Map Grid for Data Analysis and Archival, *Journal of Climate and Applied Meteorology*, **23**:1253-1257.